Generalization of the DHT mirror construction and mirror P=W conjectures

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HODGE THEORY, MIRROR SYMMETRY, AND PHYSICS OF CALABI-YAU MODULI

Goal

- Describe a generalization of Doran-Harder-Thompson (DHT) mirror construction.
- Relate two different mirror P=W conjectures:
  - Mirror for log-Calabi Yaus (Harder-Katzarkov-Przyjalkowski)
  - Degeneration/Fibration (Doran-Thompson)

Mirror Symmetry - Review (I)

- Mirror symmetry for Kähler Calabi-Yau manifolds $(X, X^\vee)$

  Complex (Algebraic) geometry of $X \longleftrightarrow$ Symplectic geometry of $X^\vee$
- The most rudimentary form of the mirror symmetry is "Hodge number symmetry"

$$h^{p,q}(X) = h^{n-p,q}(X^\vee)$$

- In 1994, M.Kontevich proposed "Homological Mirror Symmetry" as an equivalence of two categories reflecting complex geometry of X and symplectic geometry of $X^\vee$.

$$D^b Coh(X) \longleftrightarrow Fuk(X^\vee)$$

- A.Strominger, S.T.Yau, and E.Zaslow suggested the geometric interpretation of a mirror pair, as T-duality (called SYZ construction)

Mirror Symmetry - Review (II)

For a (quasi-) Fano pair (X, D), a mirror object is a Landau-Ginzburg model $(Y, \omega, w : Y \to \mathbb{C})$ where (Y, ω) is a Calabi-Yau Kähler manifold and $w : Y \to \mathbb{C}$ is a (holomorphic) function.

$$(X, D) \xrightarrow{SYZ} (Y, w : Y \to \mathbb{C})$$

There are three kinds of mirror pairs:

1. X and $(Y, w : Y \to \mathbb{C})$;
2. D and Y_{sm} where D is the anti-canonical divisor and Y_{sm} is a generic fiber of w;
3. U and Y where $U := X \setminus D$ is the complement.

Mirror Symmetry - Review (III)

Homological mirror symmetry conjecture has the following form:

$$\begin{array}{ccc} D^b\mathrm{Coh}(D) & \underset{i^*}{\overset{i_*}{\rightleftarrows}} & D^b\mathrm{Coh}(X) \\ \downarrow \cong & & \downarrow \cong \\ \mathrm{Fuk}(Y_{sm}) & \underset{\cap}{\overset{\cup}{\rightleftarrows}} & \mathrm{FS}(Y, w) \end{array}$$

HMS for U and Y is expected to be given by the categorical localization. Moreover, the canonical line bundle K_X corresponds to the functor ϕ_T induced by the monodromy T of w around the infinity.

Mirror Symmetry - Review (III)

Homological mirror symmetry conjecture has the following form:

$$
\begin{array}{ccccc}
-\otimes K_X|_D \circlearrowleft & D^b\mathrm{Coh}(D) & \underset{i^*}{\overset{i_*}{\rightleftarrows}} & D^b\mathrm{Coh}(X) & \circlearrowright -\otimes K_X \\
 & \downarrow \cong & & \downarrow \cong & \\
\phi_T|_{Y_{sm}} \circlearrowleft & \mathrm{Fuk}(Y_{sm}) & \underset{\cap}{\overset{\cup}{\rightleftarrows}} & \mathrm{FS}(Y, w) & \circlearrowright \phi_T
\end{array}
$$

HMS for U and Y is expected to be given by the categorical localization. Moreover, the canonical line bundle K_X corresponds to the functor ϕ_T induced by the monodromy T of w around the infinity.

DHT construction

Question

How are the mirror symmetry of Calabi-Yaus and the mirror symmetry of (quasi-)Fanos related?

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Doran-Harder-Thompson answers this question in the case of the Tyurin degeneration and brings many insightful ideas and questions.
Let $\pi : \mathfrak{X} \to \Delta$ be a degeneration of compact Kähler manifolds.

$$\begin{array}{ccccc} \mathfrak{X}_t & \subset & \mathfrak{X} & \supset & \mathfrak{X}_0 \\ \downarrow & & \downarrow & & \downarrow \\ t & \in & \Delta & \ni & 0 \end{array}$$

Semistable Degeneration

- A degeneration $\pi : \mathfrak{X} \to \Delta$ is called **semistable** if the total space $\mathfrak{X}$ is smooth and the degenerate fiber $\mathfrak{X}_0$ is a simple normal crossing divisor of $\mathfrak{X}$.
- A semistable degeneration $\pi : \mathfrak{X} \to \Delta$ is of type $(N+1)$ if the dual complex of $\mathfrak{X}_0$ is of dimension N.

Let $X_c = \cup_{i=0}^{N} X_i$ be a normal crossing variety of pure dimension n. The original question studied by R.Friedman is the smoothability of X_c with a semistable degeneration. One of the key conditions is the notion of d-semistablitiy. The variety X_c is d-**semistable** if

$$\bigotimes_{i=0}^{N} I_{X_i}/I_{X_i} I_D \cong \mathcal{O}_D$$

where D is the singular locus of X_c and I_D (resp. I_{X_i}) is the ideal sheaf of I_D (resp. I_{X_i}).

In the presence of the semistable degeneration $\pi : \mathfrak{X} \to \Delta$ with $\mathfrak{X}_0 = X_c$, we have $\mathfrak{X}_0|_{X_i} \sim \mathfrak{X}_t|_{X_i} = 0$ for $t \neq 0$. It implies that in $\mathrm{Pic}(X_{ij}) \cong \mathrm{Pic}(X_{ji})$, we have the following relation

$$\begin{aligned} 0 &= \mathcal{O}(X_0 + \cdots + X_N)|_{X_i}|_{X_{ij}} \\ &= \mathcal{O}(X_i)|_{X_{ij}} \otimes \mathcal{O}(\sum_{j \neq i} X_j)|_{X_{ji}} \end{aligned}$$

The right-hand side, denoted by $N(X_{ij})$, is called the **normal class** of X_{ij}. A collection of all normal classes, the $\binom{N+1}{2}$-tuple

$$N_{X_c} := (N(X_{ij})) \in \bigoplus_{i<j} \mathrm{Pic}(X_{ij})$$

vanishes if and only if X_c is d-semistable.

Smoothability

Theorem (Kawamata-Namikawa)

Let $X_c = \bigcup X_i$ be a compact Kähler normal crossing variety of dimension n such that

1. *X_c is d-semistable;*
2. *its dualizing sheaf ω_{X_c} is trivial;*
3. *$H^{n-2}(X_c, \mathcal{O}_{X_c}) = 0$ and $H^{n-1}(X_i, \mathcal{O}_{X_i}) = 0$ for all i.*

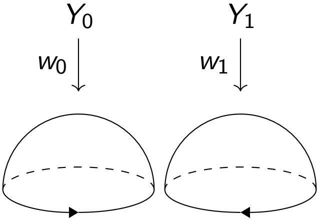
Then X_c is smoothable to a Calabi-Yau n-fold X with a smooth total space.

Definition

Let X_c be a Calabi-Yau projective normal crossing variety. X_c is called *d-semistable of type* $(N+1)$ if there exists a type $(N+1)$ semistable degeneration $\phi : \mathfrak{X} \to \Delta$ whose central fiber $\mathfrak{X}_0$ is X_c.

DHT construction

Let's consider $N = 1$ case, which is so-called Tyurin dgeneration. The degeneration fiber has two irreducible components (X_0, X_{01}) and (X_1, X_{10}). The d-semistabe condition is given by $K_{X_0}|_{X_{01}} \otimes K_{X_1}|_{X_{10}} = 0$. Suppose we have two LG models $(Y_0, \mathrm{w}_0 : Y_0 \to \mathbb{D})$ and $(Y_1, \mathrm{w}_1 : Y_1 \to \mathbb{D})$ mirror to each component such that generic fibers of them are topologically the same. The d-semistability condition can be interpreted as the monodromy condition $T_0 \circ T_1 = id$. Hence we can "topologically" glue two LG models to produce a topological fibration $\pi : Y \to S^2$.



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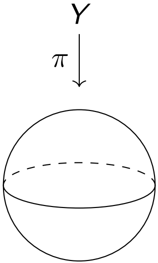

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Theorem (Doran-Harder-Thompson)

Y is a topological mirror to a smoothing X of $X_0 \cup X_1$. In other words, we have $\chi(Y) = (-1)^n \chi(X)$

Question

How to generalize this construction beyond the Tyurin degeneration case?

Extended Fano/LG Correspondence

From the SYZ perspective, a LG potential $w : Y \to \mathbb{C}$ is given by the sum of holomorphic disc counts that touches at each irreducible components. This suggests that one should think of a "multi-potential" analogue of the LG model to understand mirror symmetry for the pair (X, D) when D has more than one component. For example, let $D = D_1 \cup D_2$ and assume that the intersection $D_{12} := D_1 \cap D_2$ is smooth. Then we consider $(Y, h = (h_1, h_2) : Y \to \mathbb{C}^2)$

$$\begin{aligned} X &\longleftrightarrow (Y, w := h_1 + h_2 : Y \to \mathbb{C}) \\ D = D_1 + D_2 &\longleftrightarrow Y_{sm} = w^{-1}(t) \\ U = X \setminus D &\longleftrightarrow Y \\ (D_1, D_{12}) &\longleftrightarrow (Y_1, h_2|_{Y_1} : Y_1 \to \mathbb{C}), Y_1 = h_1^{-1}(t_1) \\ (D_2, D_{12}) &\longleftrightarrow (Y_2, h_1|_{Y_2} : Y_2 \to \mathbb{C}), Y_2 = h_2^{-1}(t_2) \\ D_{12} &\longleftrightarrow Y_{12} \end{aligned}$$

where $h : Y \to \mathbb{C}^2$ is flat and proper and Y_{12} is Calabi-Yau.

Extended Fano/LG Correspondence

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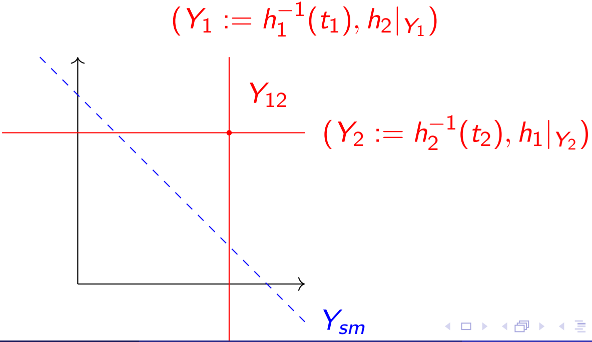

Hybrid/Higher rank Landau-Ginzburg model

Definition

A **hybrid/higher rank Landau-Ginzburg (LG) model of rank 2** is a triple $(Y, \omega, h = (h_1, h_2) : Y \to \mathbb{C}^2)$ where

1. (Y, ω) is a n-dimensional complex Kähler Calabi-Yau manifold with a Kähler form $\omega \in \Omega^2(Y)$;
2. $h : Y \to \mathbb{C}^2$ is a proper surjective holomorphic map such that
    1. (Local trivialization) there exists a constant $R > 0$ such that
        - $h : Y \to \mathbb{C}^2$ is a locally trivial symplectic fibration over $B_{12} := \{|z_i| > R | i = 1, 2\} \subset \mathbb{C}^2$;
        - the map $h_i : Y \to \mathbb{C}_{z_i}$ is a locally trivial symplectic fibration over $B_i := \{|z_i| > R\} \subset \mathbb{C}_{z_i}$ for $i = 1, 2$;
        - for $\{i, j\} = \{1, 2\}$, over B_i, we have $v(h_j) = 0$ for any horizontal vector field $v \in T^{h_i}Y$ associated to h_i.
    2. (Compatibility) for $i = 1, 2$, such local trivializations are compatible under the natural inclusions $B_{12} \subset B_i \times \mathbb{C}$.

Hybrid/Higher rank Landau-Ginzburg model

Definition

A **hybrid/higher rank Landau-Ginzburg (LG) model of rank** N is a triple $(Y, \omega, h = (h_1, h_2, \ldots, h_N) : Y \to \mathbb{C}^N)$ where

1. (Y, ω) is n-dimensional complex Kähler Calabi-Yau manifold with a Kähler form $\omega \in \Omega^2(Y)$;
2. $h : Y \to \mathbb{C}^N$ is a proper surjective holomorphic map such that
   1. (Local trivialization) there exists a constant $R > 0$ such that for any non-empty subset $I \subset \{1, \ldots, N\}$, the induced map $h_I : Y \to \mathbb{C}^{|I|}$ is a locally trivial symplectic fibration over the region $B_I := \{|z_i| > R | i \in I\}$ with smooth fibers. Furthermore, over B_I we have $v(h_j) = 0$ for any horizontal vector field $v \in T^h Y$ and $j \notin I$;
   2. (Compatibility) for $I \subset J$, such local trivializations are compatible under the natural inclusions $B_J \times \mathbb{C}^{N-|J|} \subset B_I \times \mathbb{C}^{N-|I|} \subset \mathbb{C}^N$.

Extended Fano/LG Correspondence

1. $(X, D) = (\mathbb{P}^2, L_1 \cup L_2 \cup L_3)$. Then $(Y, h : Y \to \mathbb{C}^3)$ is given by a Hori-Vafa mirror

$$((\mathbb{C}^*)^2, h(x,y) = (x, y, \frac{1}{xy}))$$

2. $(X, D) = (\mathbb{P}^2, L \cup C)$ Then $h : Y \to \mathbb{C}^2$ is a branched double cover of $\mathbb{C}^2$ whose discriminant locus is

$$\{a^2 b = 4\} \cup \{b = 0\}$$

One can check the mirror relations: $(Y_2 = \{b : \text{const}\}, h_1|_{Y_2} : Y_2 \to \mathbb{C})$ is mirror to $(H = \mathbb{P}^1, H \cap C = 2\text{pts})$, $w := h_1 + h_2 : Y \to \mathbb{C}$ elliptic fiber with two points removed.

Proposition (Gluing property)

Let $(Y, \omega, h : Y \to \mathbb{C}^N)$ be a hybrid LG model and H be a generic hyperplane in the base $\mathbb{C}^N$, which is not parallel to any coordinate hyperplanes. There exists an open cover $\{U_i\}_{i=1}^N$ of H such that for any non-empty subset $I \subset \{1, \dots, N\}$, the induced map $h^{-1}(U_I) \to U_I$ is symplectic isotopic to the induced hybrid LG potential $h_{Y_I} : Y_I \to \mathbb{C}^{N-|I|}$ which is linear along the base.

There are N monodromy operations induced by taking a loop T_i near the infinity on the base of $h : Y \to \mathbb{C}^N$

$$T_i := (t_1, \dots, t_{i-1}, e^{\sqrt{-1}\theta} t_i, t_{i+1}, \dots, t_N) \quad (0 \leq \theta \leq 2\pi)$$

for a generic $(t_1, \dots, t_N) \in \mathbb{C}^N$ and $i = 1, \dots, N$. The monodromy T_i induces not only the automorphism of a generic fiber $Y_i = h_i^{-1}(t)$ but also the automorphism of the induced fibration $h|_{Y_j} : Y_j \to \mathbb{C}^{N-1}$ for any i, j.

Definition

Let $(X, D = \cup_{i=1}^{N} D_i)$ be a (quasi) Fano pair. A hybrid LG model $(Y, \omega, h : Y \to \mathbb{C}^N)$ is mirror to (X, D) if it satisfies mirror relations

$$(D_I, D(I)) \longleftrightarrow (Y_I, \omega_{Y_I}, h|_{Y_I} : Y_I \to \mathbb{C}^{N-|I|})$$

for all I. Here $D_I = \cap_{i \in I} D_i$, $D(I) = \sum_{i \notin I} D_i \cap D_I$ and $Y_I = \cap_{i \in I} Y_i$.

Homological mirror symmetry (HMS) should be stated as an isomorphism of cubical diagrams of categories. However, the challenge is in the A-side.

Ansatz (line bundle/monodromy correspondence)

For such mirror pair $\{(X, D)|(Y, h)\}$, there is a correspondence between line bundles and monodromies

$$\mathcal{O}_X(D_i) \longleftrightarrow T_i$$

for $i = 1, \ldots, N$. In particular, we have $-K_X \longleftrightarrow T := T_1 \circ \cdots \circ T_N$

Mirror Construction - Higher Type

For simplicity, let $N = 2$. Consider a semistable degeneration of Calabi-Yau n-folds $\pi : \mathfrak{X} \to \Delta$ where $\mathfrak{X}_0 = X_0 \cup X_1 \cup X_2$. We will construct a topological mirror of the smoothing X. For $\{i, j, k\} = \{0, 1, 2\}$, since X_i comes with a canonical choice of the anti-canonical divisor $X_{ij} \cup X_{ik}$, we should start with hybrid LG model $(Y_i, \omega_i, h_i = (h_{ij}, h_{ik}) : Y_i \to \mathbb{D}^2)$ as a mirror of $(X_i, X_{ij} \cup X_{ik})$. Note that there should be some topological constraints because we have already identified divisors of X_i's. The d-semistability condition is given by

$$\mathcal{O}(X_i)|_{X_{ij}} \otimes \mathcal{O}(X_{ji} + X_{jk})|_{X_{ji}} = 0$$

Mirror counterpart is given by

$$\mathcal{O}(X_i)|_{X_{ij}} \longleftrightarrow T_{ij} \in Aut(Y_{ij}, h_{ik})$$
$$\mathcal{O}(X_{ji} + X_{jk})|_{X_{ji}} \longleftrightarrow T_j = T_{ji} \circ T_{jk} \in Aut(Y_{ji}, h_{jk})$$

Recall that $\mathbb{P}^2$ has a trisection of three polydisks: $\mathbb{D}^2 \cong Z_i = \{|z_j|, |z_k| \le |z_i|\} \cong\subset \mathbb{P}^2$. One can write down the gluing condition of these polydisks along the boundaries in terms of matching of the loops. By regarding them as bases of the hybrid LG models, this gluing condition is the same with the monodromy conditions. Due to the topological constraints, we obtain a topological fibration $f : Y \to \mathbb{P}^2$. We claim that this is a topological mirror of X.

Theorem (L.)

Let $X_c = \cup_{i=0}^N X_i$ be a d-semistable Calabi-Yau projective normal crossing variety of type $(N+1)$. Suppose that we have hybrid mirror LG models $(Y_i, \omega_i, h_i : Y \to \mathbb{C}^N)$ for X_i's with topological constraints. Then these hybrid LG models are glued to be a fibration $f : Y \to \mathbb{P}^N$ which is topological mirror to a smoothing of X_c.

Remarks

- There is another direction of generalization for a semistable degeneration of Fano manifolds (Doran-Fenglong-Kostiuk).
- Note that this construction is purely topological so that it is not plausible to study relations of other versions of mirror symmetry (e.g. Hodge numbers). See the works of A.Kanazawa/Barrott-Doran for the gluing in the complex category.
- On the degeneration side, there is a monodromy action on the smoothing Calabi-Yau X, which yields the monodromy weight fitration on the cohomology $H^*(X, \mathbb{C})$. A natural question is to find the corresponding filtration on the mirror $H^*(Y)$. It turns out to be the perverse Leray filtration on $H^*(Y)$ associated to the fibration $\pi : Y \to \mathbb{P}^N$, and the precise formulation and evidences are given by Doran-Thompson.

Perverse Filtration

Let $f : Y \to B$ be a morphism between algebraic varieties.

$$H^k(Y;\mathbb{C}) = \mathbb{H}^k(B, Rf_*\mathbb{C})$$

The perverse Leray filtration $P^f_\bullet$ on $H^\bullet(Y)$ is defined as

$$P^f_b H^k(Y,\mathbb{C}) := \mathrm{Im}(H^k(B,{}^p\tau_{\leq b} Rf_*\mathbb{C}) \to H^k(B, Rf_*\mathbb{C}))$$

where ${}^p\tau$ is truncation associated to the perverse t-structure on $D^b_c(B)$.

Consider a pair of general flags $(B_\bullet, B'_\bullet)$ of B

$$\cdots \subset B_j \subset B_{j-1} \subset \cdots \subset B_0 = B$$
$$\cdots \subset B'_j \subset B'_{j-1} \subset \cdots \subset B'_0 = B$$

and let $Y_j = f^{-1}(B_j)$ and $Y'_j = f^{-1}(B'_j)$.

Theorem (Cataldo-Migliorini)

1. If B is affine, then the perverse filtration is given by

$$P_b H^k(Y, \mathbb{C}) = \mathrm{Ker}(H^k(Y, \mathbb{C}) \to H^k(Y_{b-k+1}, \mathbb{C}|_{Y_{b-k+1}}))$$

2. If B is quasi-projective, then the perverse filtration is given by

$$P_b H^k(Y, \mathbb{C}) = \mathrm{Im}\{\bigoplus_{i+j=b-k} \mathbb{H}^k_{Y'_{-j}}(Y, \mathbb{C}|_{Y-Y_{i-1}}) \to \mathbb{H}^k(Y, \mathbb{C})\}$$

Mirror P=W conjecture

Combining with the Deligne's canonical mixed Hodge structure, we define

Definition (Perverse-Mixed Hodge Polynomial)

For any (non-singular) quasi-projective variety U, we define a perverse-mixed Hodge polynomial as follows

$$PW_U(u,t,w,p) = \sum_{a,b,r,s} (\dim \mathrm{Gr}_F^a \mathrm{Gr}^W_{s+b} \mathrm{Gr}^P_{s+r}(H^s(U,\mathbb{C}))u^a t^s w^b p^r$$

where $P_\bullet$ corresponds to the affinization map $\mathrm{Aff}: U \to \mathrm{Spec}(\Gamma(\mathcal{O}_U))$

Mirror P=W Conjecture [2018] (Harder-Katzarkov-Przyjalkowski)

Let $(U, U^\vee)$ be mirror log Calabi-Yau pairs of dimension n. Then the perverse-mixed Hodge polynomials of U and $U^\vee$ satisfy the following relation.

$$PW_U(u^{-1}t^{-2}, t, p, w)u^n t^n = PW_{U^\vee}(u,t,w,p)$$

- Set $w = 1$ and change variables, we have a simplified form of the conjecture

$$\dim \bigoplus_{p} \mathrm{Gr}_F^p \mathrm{Gr}^W_{p+q+r} H^{p+q}(U) = \dim \mathrm{Gr}^P_{n-p+q+r} H^{d-p+q}(U^\vee)$$

- In general, for the computation, we should choose a compactification (X, D) of U and a proper affinization map $h : U^\vee \to \mathbb{C}^N$ and look at the spectral sequences.
- We expect that mirror symmetry of such extra data $\{(X, D), (U^\vee, h)\}$ lifts the mirror P=W statement as an isomorphism of the spectral sequences: We call it a *cohomological mirror P=W conjecture*

Theorem (L.)

Given a hybrid LG model $(U^\vee, \omega, h : U^\vee \to \mathbb{C}^N)$*, there is a well-defined cubical diagram of cohomology groups*

$$H^{k-N}(U^\vee\{N\}) \xrightarrow{\psi_N} \bigoplus_{|I|=N-1} H^{k-N+1}(U_I^\vee, U_{I,sm}^\vee) \xrightarrow{\psi_{N-1}} \cdots$$

$$\xrightarrow{\psi_2} \bigoplus_{|I|=1} H^{k-1}(U_I^\vee, U_{I,sm}^\vee) \xrightarrow{\psi_1} H^k(U^\vee, U_{sm}^\vee)$$

It recovers the E_1*-spectral sequence for the perverse Leray filtration associated to* $h : U^\vee \to \mathbb{C}^N$ *on* $H^*(U^\vee)$*.*

This theorem allows one to state the cohomological lift of the mirror P=W conjecture as an isomorphism of two spectral sequences. In other words, one can interpret the mirror P=W as a result of the functoriality in the extended Fano/LG correspondence.

Combing back to our degeneration story, let X be a smoothing Calabi-Yau and $\pi : Y \to \mathbb{P}^N$ be its topological mirror.

Conjecture (Doran-Thompson)

For $p, q, l \geq 0$, there is an isomorphism of cohomology groups

$$\bigoplus_{q-p=a} \mathrm{Gr}_F^p \mathrm{Gr}_{l+2p-n}^{W_{lim}} H^{p+q}(X) \cong \mathrm{Gr}_l^P H^{n+a}(Y)$$

where $W_\bullet^{\lim}$ is the monodromy weight filtration on $H^*(X)$ and $P_\bullet$ is the perverse filtration associated to $f : Y \to \mathbb{P}^N$ on $H^*(Y)$.

Theorem(L.)

Suppose that the cohomological P=W conjecture for mirror pairs $(X_i, \sum_{j\neq i} X_j), (Y_i, \omega_i, h_i : Y_i \to \mathbb{C}^N)$ hold for all i. Then the above conjecture holds.

Thank you!!