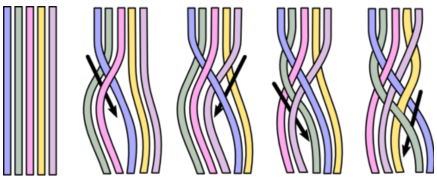
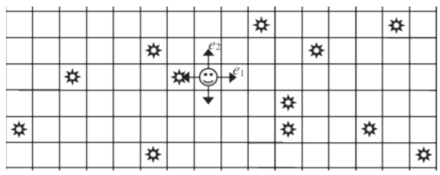
Seminar für Master–Studierende

Im Kuriositätenkabinett der Gruppen

Prof. Dr. Petra Schwer[1] und Dr. José Pedro Quintanilha[2]



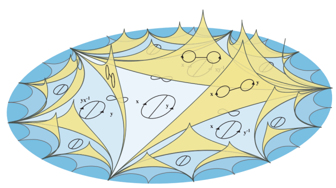


Wie wild kann eine Gruppe sein?

Dieses Seminar ist eine geführte Tour durch einige der faszinierendsten und ungewöhnlichsten Beispiele der geometrischen Gruppentheorie.
Im Mittelpunkt stehen Gruppen mit überraschenden Konstruktionen, exotischen Wirkungen oder unerwarteten algebraischen Eigenschaften. Beispiele sind Thompsons Gruppen, Zopfgruppen, sowie Gruppen, die auf Graphen, nichtpositiv gekrümmten oder fraktalen Räumen wirken.

Wir bieten einen Einstieg in verschiedenste aktuelle Themen und schlagen Brücken zur Forschung. Vorkenntnisse über Geometrische Gruppentheorie oder Topologie sind für einzelne der Vortragsthemen nötig.

- Für Bachelor und Master
- Blockkurs and 4 Nachmittagen
- Details siehe Webseite



`https://web.mathi.uni-heidelberg.de/ggt/ss26-groups`

Vorbesprechung
Dienstag 14. April 2026, 16 Uhr (c.t.)
Raum wird noch bekanntgegeben

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A cabinet of curiosities of groups, Summer 2026

Petra Schwer, José Quintanilha

Topics for talks

All talks belong to (at least) one of the following categories:

A Groups with interesting constructions

B Groups coming from interesting geometric actions

C Groups with interesting algebraic or algorithmic properties

Please note: The content descriptions below should give a rough idea of the topic and provide a minimal requirement for each talk. It is subject to the speaker to make final choices of the outline and content of the talk.

1. **RAAGs** (C, also B) Right-angled Artin groups are a class of groups defined by means of explicit, simple presentations encoded in a finite graph. How much does the structure of the graphs determine the group itself and the collection of its subgroups? What about algorithmic properties of these groups? Are there nice spaces they act on?

   Sources: [CM17] Chapter 14 (Skip Section 14.2) and [S23] Section 4.4.

2. **Lamplighter groups** (A, C) Explain what a Lamplighter group is and how it relates to Diestel-Leader graphs. Explain why they are special instances of wreath products. Lamplighter groups have dead ends - that is vertices in the Cayley graph where you can not get further away from the identity. How frequent are they?

   Sources: [CM17] Chapter 15 and [CT05] or Section 4 in [BDP18]

3. **Automorphisms of free groups** (B,C) Free groups can be seen as fundamental groups of roses with n petals and their automorphism groups have surprisingly simple combinatorial descriptions in terms of combinatorial data on these roses. Explain the concept of a train track map and how to associate one to an automorphism of a free group. Explain Theorem 1.7 and the outline of its proof.

   Sources: [BH92] and [CM17] Chaper 6

4. **Braid groups and non-crossing partitions (NCPs)** (A,B) Braid groups have many interesting equivalent definitions and presentations. One way to define them is via so called braid diagrams. In this talk we will also get to know a partial order on symmetric groups which allows for a new presentation. The talk should also explain the connection

between braid groups and NCPs - yet another combinatorial object related to symmetric groups.

Source: [B01]

5. **Mapping class groups** (B) Explain what a Mapping class group is and introduce Dehn twists. Prove that mapping class groups of S_g are generated by $2g + 1$ Dehn twists. What is the connection to braid groups? Can you name open problems?

   Sources: [CM17] Chapter 17, [FM11] e.g. Section 2.2 for a concrete simple example.

6. **Out(F_n) and Outer space** (A,B) Free groups and their automorphisms have been of interest for a very long time. Only recently though, a space, called outer space, has been constructed that allows to geometrically study properties of the outer automorphisms of free groups. Define the outer space. Why is this space an interesting and useful object to study $Out(F_n)$? E.g. state main theorem of [BF01].

   Sources: [V08], Vogtmann has many surveys and lecture notes on Outer space. See for example Part I of [V02] or the lecture notes [V17]. Another possible reference is [B12]. (Pick one of the last three).

7. **Thompson's group F** (A,C) Thompson's group F is one of the most prominent groups in GGT. *„It has a number of seemingly paradoxical behaviors.“* Sean Cleary writes in [CM17]. For example, F contains a direct sum of countably infinite copies of itself. We will learn the construction as well as some basic properties of F in this talk. Give an outline of the proof of Cor. 2.6 in [CFP09].

   Sources: [CM17] Chapter 16 (skip Section 16.5) and [CFP09] Sections 1 and 2 or [BDP18] Section 2; see also [CF11] for a quick intro

8. **Thompson's group F and its cousins**(A,C) First we learn more about the cool properties the group F has. Moreover, F has two cousins: the groups V and T. How are those constructed? (Choose one of them) How do they differ from or share properties with F?

   Sources: [CM17] Chapter 16.5 and [CFP09] Sections 4-6, see also [CF11] for a quick intro to F

9. **Automatic groups** (C) The word problem is closely linked to computational aspects in group theory and also to several formal concepts in theoretical computer science. In this talk we get to know groups describable by finite state automata. Explain the definition and highlight some basic properties of automatic groups.

   Sources: Start with [R22] and Sections 5.1 to 5.5 in [HRR17]. See also Section 3 in [BDP18] or, as alternative sources, [Sch25] or [Cho02].

10. **Baumslag–Solitar groups** (C) In 1962, Baumslag and Solitar introduced a two-parameter family of groups $\mathrm{BS}(m, n)$ (with $m, n \in \mathbb{Z}\backslash\{0\}$), among which are counterexamples to a conjecture which predicted that two-generated finitely presented groups are "Hopfian". This talk introduces the Baumslag–Solitar groups and the Hopfian property. The goal is to give a partial proof of Baumslag–Solitar's main result, namely [Bog08, Theorem II.29.1] and [Bog08, Corollary II.29.12]. This will require recalling HNN extensions and their normal forms [Bog08, Section II.14], as well as a explaining the property of being residually finite. The speaker may also state the full characterization in Baumslag–Solitar's original article [BS62, Theorem 1].

    Sources: [Bog08, Chapter II, Sections 14 and 29]. A gentle introduction to Baumslag–Solitar groups is given in [BDP18, Section 5]. The original paper is [BS62].

11. **Knot groups** (A,C) The most important algebraic invariant of a 3-manifold is by far its fundamental group. In particular, if one wishes to understand a knot $K \subset \mathbb{S}^3$, it is fruitful to study the fundamental group $\pi_1(\mathbb{S}^3\backslash K)$ of its complement. This talk introduces knots (which Rolfsen calls "tame knots"), their diagrams, and sketches the construction of the Wirtinger presentation following [Rol90, Chapter 3]. The speaker is encouraged to present examples or other facts (not necessarily with proof), another suitable source for which is [Lic97, Chapter 11]. This talk should best be reserved for speakers with some background in algebraic topology.

    Sources: [Rol90] Chapter 3 and [Lic97] Chapter 11

12. **TDLC groups** (B,C, new viewpoint on groups)

    Nature often supplies us with groups endowed with additional structure, such as a topology compatible with the group operations, or even the structure of a differentiable manifold. A class of topological groups that has been gaining increasing attention are totally disconnected locally compact (TDLC) groups (which include the abstract groups, equipped with the discrete topology). This talk will briefly introduce topological groups, define and give examples of profinite groups, and present one of the fundamental structural theorems in the theory of TDLC groups: Van Dantzig's Theorem. If time permits, it would also be interesting to see some examples of (non-profinite) TDLC groups.

    Sources: Mainly [Cas20] up to Section 3. The in-progress book manuscript [Kra, Chapter 1] also has an account of Van Dantzig's Theorem.

Organisational comments

The seminar will take place as a partially blocked course. Please keep the following dates reserved for the seminar in case you would like to participate:

Fridays, May 15th and 22nd, July 17th and 24th.

The tentative schedule for each of these meetings is:

13:15 - 14:30 first talk
15:00 - 16:15 second talk
16:30 - 17:45 third talk

Please note the following:

- We require all participants to have a strong background in GGT, e.g. on the level of the lecture *Geometrische Gruppentheorie I.*
- All talks are on a master level. Advanced Bachelor students may also participate. We need to figure out how this can count towards your credits.
- Upon succesful participation we can talk about potential Master thesis topics.
- A mandatory preliminary meeting with either José Quintanilha or Petra Schwer should happen at the latest 10 days prior to your scheduled talk. Please contact us early enough to schedule such a meeting.
- We expect you to prepare the outline of the talk based on the listed material in time for the aforementioned preliminary meeting.
- We expect you to participate on all block days of the course even if your talk has already happened.
- Each talk will last 70mins including questions. A handout of 1 page (A4, font size 11) should be provided.
- Please register on Müsli.

References

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[Bog08] Oleg Bogopolsi *Introduction to group theory.* translated from the Russian, with a new chapter, EMS Textbooks in Mathematics (2008) `https://doi.org/10.4171/041`

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[CT05] Sean Cleary and Jennifer Taback, *Dead end words in lamplighter groups and other wreath products*, Q.J. Math. 56, No. 2, (2005)

[FM11] Benson Farb and Dan Margalit, *A Primer on Mapping Class Groups*, Princeton University Press, (2011). https://doi.org/10.1515/9781400839049

[HRR17] Derek Holt, Sarah Rees and C.E. Röver, *Groups, Languages and Automata*, LMS Student Texts 88, Cambridge University Press. (2017).

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[Lic97] W.B. Raymond Lickorish, *An Introduction to Knot Theory*, Graduate Texts in Mathematics, vol. 175, Springer (1997).

[M08] John Meier, *Groups, Graphs and Trees*, Cambridge University Press, 2008.

[R22] Sarah Rees, *The development of the theory of automatic groups* (2022). https://arxiv.org/pdf/2205.14911

[Rol90] Dale Rolfsen, *Knots and Links*, 2nd print. with corr., AMS Chelsea Publishing (1990).

[Sch25] Eduard Schesler, *Mini-course on automata and groups* https://eduardschesler.de/Talks_automata_and_groups%20part%201.pdf

[S23] Petra Schwer, *CAT(0) cube complexes. An introduction*, Springer (2023).

[V08] Karen Vogtmann, *What is Outer space?*, AMS Notices, (2008) https://www.ams.org/notices/200807/tx080700784p.pdf

[V02] Karen Vogtmann, *Automorphisms of Free Groups and Outer Space*, Geometriae Dedicata 94, 1–31 (2002). https://doi.org/10.1023/A:1020973910646

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