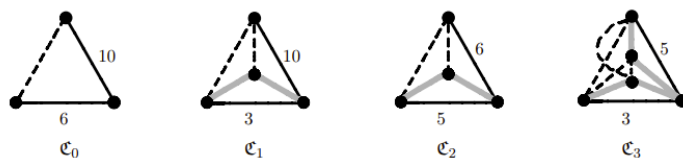


Seminar for Master–Students

The isomorphism problem for Coxeter groups



Prof. Dr. Petra Schwer¹ and Dr. José Pedro Quintanilha²

The isomorphism problem for Coxeter groups raises the fundamental question of when two Coxeter groups, described by different graphs, actually possess the same abstract group structure.

This seminar explores both the history of the problem and results relating to the very latest developments from the Summer of 2026. We will gain insight into the connection between algebraic and geometric structures in the context of Coxeter groups.

We shall see how defining graphs can be manipulated without altering the abstract group structure. Conclusions about the group itself can also be drawn from its finite quotients. Such profinite rigidity properties also have implications for the isomorphism problem.

Preliminary meeting

Monday 12. October 2026, 13:00 (sharp) in SR3.

Register for the seminar on *Special topics in GGT* in HeiCo!



- For students of the Master's in Mathematics
- Block seminar in 4 afternoons
- See the website for details:

<https://web.mathi.uni-heidelberg.de/ggt/ws26-ipcg>

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The isomorphism problem for Coxeter groups

Seminar, Winter semester 2026

Organizers: Petra Schwer, José Pedro Quintanilha

The material naturally splits into four parts. The titles and abstracts for the talks are listed below.

Please note: The content descriptions below should give a rough idea of the topic and provide a minimal requirement for each talk. It is subject to the speaker to make final choices of the outline and content of the talk. Contact us early on in case you are having trouble making a selection.

Setting the stage and early results

1. What's the problem?

This is an overview talk on the isomorphism problem that also covers the very basic definitions of Coxeter systems. Following the survey of Santos-Rego-Schwer [SS24], Coxeter systems are organized as the vertices of a simplicial complex called the galaxy of Coxeter groups, which will be introduced to set the stage for the seminar.

2. Strong rigidity for certain geometric actions

A result of Charney–Davis [CD00] establishes that Coxeter groups with geometric actions on contractible (pseudo-)manifolds are *strongly rigid*, which means that any two standard generating sets are conjugate. This talk explains the proof of this result.

3. Reflections in abstract Coxeter groups

Can one recognize reflections from the abstract group structure, without knowing the Coxeter generating set? Franzsen–Howlett–Mühlherr [FHM06] introduced the notion of the *finite continuation* of a finite order element in a group, and showed that for irreducible Coxeter systems (W, S) with W infinite and all parameters $m_{s,t}$ finite ($s, t \in S$), reflections are intrinsically describable as the involutions that span their own finite continuation. This establishes strong rigidity for this class of Coxeter groups.

Vertically navigating the galaxy

4. Vertical edges decompose into blow-ups

Given a Coxeter system (W, S) , Mihalik–Ratcliffe–Tschantz [MRT07] introduce the notion of a *basic subset* of S , and show that given two Coxeter generating sets S, S' for W , there is a canonical one-to-one correspondence (“matching”) between the basic subsets of S and of S' .

It turns out that the existence of matching subsets spanning non-isomorphic subgroups allows one to perform a “blow up” operation that increases the rank of one of the given Coxeter systems. Drawing from this work one can see that blow-ups are “atomic vertical edges”, in the sense that they essentially suffice to describe modifications of defining graphs that increase the rank. The talk should explain Theorems 7.7 and 9.1 in [MRT07].

Horizontally navigating the galaxy

5. Twisting diagrams

A particularly interesting early example of a nonrigid Coxeter group has been given by Bernhard Mühlherr. Generalizing this construction, Brady–McCammond–Mühlherr–Neumann introduced an operation on Coxeter systems called a *twist* [BMMN02]. In this talk we will learn that two Coxeter groups defined by diagrams that differ by a twist are isomorphic. (Ignore sections 7 and 8 of the paper.)

6. The twist conjecture

There are several ways of manipulating a Coxeter diagram and obtain a new diagram of the same rank, and whose group is isomorphic to the original one. The twist conjecture however states that “essentially” twists are enough. This talk explains the precise statement of the conjecture and how it implies a solution to the isomorphism problem. If time permits, explain the history of the conjecture and highlight the recent breakthrough by Fioravanti [Fio26].

Sources: [SS24, Setions 5.1 - 5.3], [Fio26, Appendix A].

7. The twist conjecture for skew-angled Coxeter groups

This conceptual ancestor of the twist conjecture considers the special case of Coxeter systems that are *skew-angled*, meaning there are no graph labels $m_{s,t} = 2$. This talk explains the main result of Mühlherr–Weidmann [MW02] and how it relates to the general twist conjecture (as introduced in the previous talk). Based on this result it was only natural to ask for a similar statement in the class of all Coxeter systems.

8. The twist conjecture for chordal Coxeter groups

A graph is *chordal* if every cycle of length at least four has a chord. For a defining graph of a Coxeter group the property of being chordal is independent of a chosen generating set. Ratcliffe and Tschantz prove the twist conjecture for chordal Coxeter groups [RT08]. Cover at least Theorem 8.4 and 9.2 (including proofs) and the material leading up to them. If time permits explain the angle deformation theorem.

Profinite rigidity

9. What is profinite rigidity?

The profinite completion of a finitely generated group G is a totally disconnected compact group $\widehat{G}$ that packages all the information about the finite quotients of G . An active area of current research is the question of when two nonisomorphic groups can have isomorphic profinite completions. This talk follows Alan Reid’s ICM survey [Rei18] introducing the notions of residual finiteness, profinite completions, and the question of profinite rigidity (both absolute and relative to various classes of groups). Explain the proof of relative profinite rigidity of abelian groups (Proposition 3.1) and also how profinite rigidity yields an algorithm for solving the isomorphism problem [BCR16, p.5]. Feel free to give other examples you find interesting.

Much of the theory is built around fundamental groups of 3-manifolds, so if you wish to present something in this direction, an up-to-date overview may be found, for example, in the introduction to Xiaoyu’s Xu recent paper [Xu25].

10. Profinite rigidity and the isomorphism problem in small rank

This talk illustrates an early example for how profinite rigidity can be used to solve the isomorphism problem. Santos-Rego–Schwer show that two Coxeter groups of rank ≤ 3 are isomorphic if and only if their profinite completions are isomorphic [SS24, Section 4]. For triangle groups this has the following consequence: the multiset of the three edge labels of the defining graph determines is determined by the isomorphism type.

11. Profinite rigidity of affine Coxeter groups

This talk explains Möller–Varghese’s proof that irreducible affine Coxeter groups are profinitely rigid among Coxeter groups [MV24]. A key step is showing that these are precisely the Coxeter groups that are “just infinite”. Mention also (without proof) the upgrade to this result due to Corson–Hughes–Möller–Varghese [CHMV24].

Organisational comments

The seminar will take place as a partially blocked course. Please keep the following dates reserved for the seminar in case you would like to participate:

Fridays, November 6 and 13, December 11, January 22.

The tentative schedule for each of these meetings is:

13:15 - 14:30 first talk

15:00 - 16:15 second talk

16:30 - 17:45 third talk

Please note the following:

- We require all participants to have a strong background in GGT, e.g. on the level of the lecture *Geometrische Gruppentheorie I*. Preliminary knowledge on Coxeter groups is very helpful but not mandatory for participation. Talk to us in case you are in doubt.
- All talks are on a master level. Advanced Bachelor students may also participate. We then would need to figure out how this can count towards your credits.
- Upon successful participation we can talk about potential Master thesis topics.
- A mandatory preliminary meeting with either José Pedro Quintanilha or Petra Schwer should happen at the latest 10 days prior to your scheduled talk. Please allow for a few days and contact us early enough to schedule such a meeting.
- We expect you to prepare the outline of the talk based on the listed material in time for the aforementioned preliminary meeting.
- We expect you to participate on all block days of the course even if your talk has already happened.
- Each talk will last 70mins including questions. A handout of 1 page (A4, font size 11) summarizing the main points of the talk should be provided.
- One of the primary goals of the seminar is that students learn to independently select, organize and present mathematical material from the literature. We therefore expect the structure and contents of the talk, as well as the handout, to be authored by the speakers. Usage of LLMs (if any) should be restricted to minor tasks, such as help with translation or with $\text{\LaTeX}$ code.
- Please register for the seminar on *Special topics in GGT* on HeiCo.

References

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- [Xu25] X. Xu, *Profinite almost rigidity in 3-manifolds*, Adv. Math 480 (2025) 110505. DOI: 10.1016/j.aim.2025.110505.